BEAM BENDING

8. Principle: **Bending moments and shear stress diagrams**

Engage:

Ride a skateboard into class.

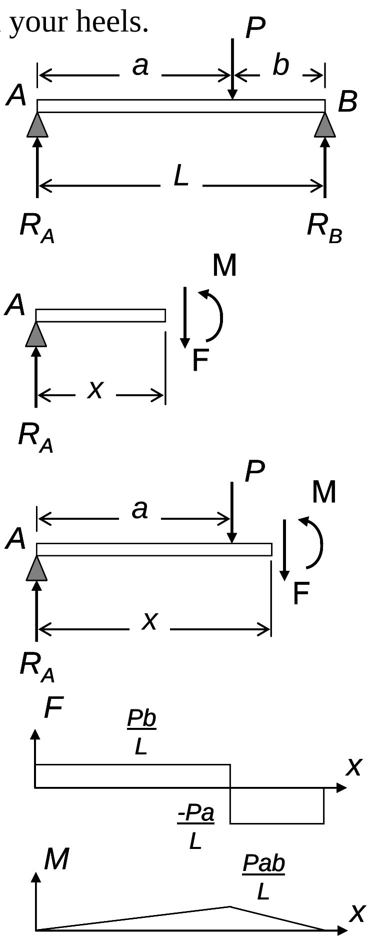
Explore:

Discuss the shear forces and bending moments set-up in the skateboard when you stand on it sideways balanced on your heels, i.e. approximating a point load. When you stand of the board more normally, how do the shear forces and bending moments change? Discuss where you need to stand to induce a zero bending moment.

You might want to ask students to work in pairs to draw schematics of these loading schemes.

Explain:

Plot the shear force and bending moment diagrams for the case where you were rocking on your heels.



Considering the complete beam

Resolve vertically: $R_A + R_B = P$

Moments about A: $Pa - R_B L = 0$

Thus: $R_A = \dfrac{Pb}{L}$ $R_B = \dfrac{Pa}{L}$

Considering the cut section (0<*x*<*a*)

Resolve vertically: $F = R_A = \dfrac{Pb}{L}$

Moments: $M = R_A x = \dfrac{Pbx}{L}$

Considering the cut section (a<*x*<*l*)

Resolve vertically:

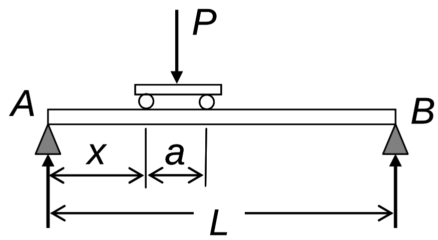
$$F = R_A - P = \frac{Pb}{L} - P = -\frac{Pa}{L}$$

Moments:

$$M = R_A x - P(x - a) = \frac{Pa}{L}(L - x)$$

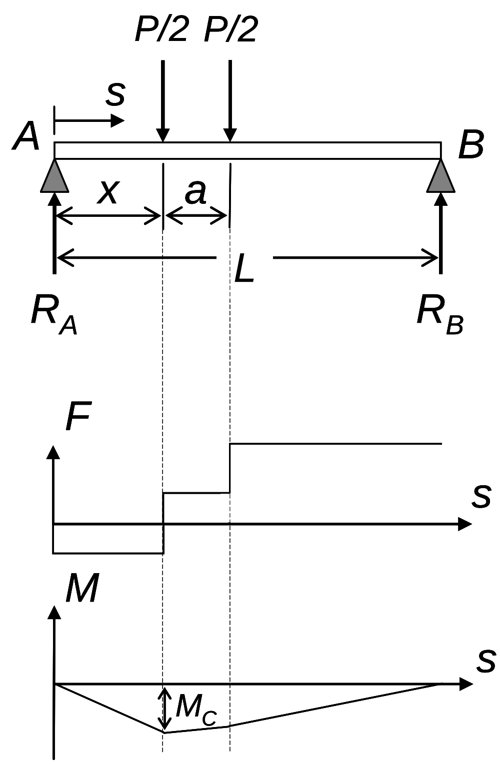
Elaborate

When a skateboarder crosses a plank we can determine the position at which the bending moment is a maximum. The situation can be idealized as shown below:



The shear force and bending moment diagrams can be plotted as previously considering small sections of beam, i.e.

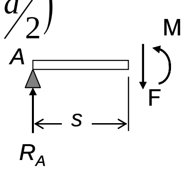
Taking moments about B gives:



$$R_A L - \frac{P}{2}(L - x - a) - \frac{P}{2}(L - x) = 0$$

Thus, $R_A = \frac{P}{L}\left(L - x - a/2\right)$

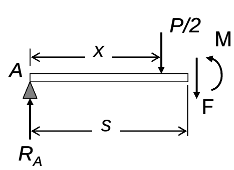
For $(s < x)$



Resolving vertically: $F = -R_A$

Taking moments: $M = -R_A s$

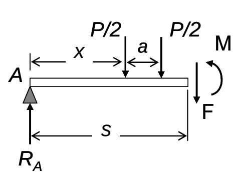
For $(x < s < (a + x))$



Resolving vertically: $F = \frac{P}{2} - R_A$

Taking moments: $M = \frac{P}{2}(s - x) - R_A s$

For $(s > (a + x))$



Resolving vertically: $F = P - R_A$

Taking moments:

$$M = P\left(s - x - \frac{a}{2}\right) - R_A s$$

N.B. These diagrams have been plotted assuming that $(x + a/2) > L/2$*, if this were not the case then the diagrams would look slight different. The symmetry of the situation allows only this case to be considered. However a There are two places where* M_c *occurs symmetric about the mid-point of the beam.*

For maximum M_C:

$$\frac{\partial M_C}{\partial x} = 0 \text{ and } M_C = R_A x = \frac{Px}{2L}(2L - 2x - a)$$

$$\text{So, } \frac{\partial M_C}{\partial x} = 2L - 4x - a = 0 \text{ and } x = \frac{L}{2} - \frac{a}{4}$$

$$\text{Thus, } \hat{M}_C = \frac{P}{2L}\left(2L - 2\left(\frac{L}{2} - \frac{a}{4}\right) - a\right)\left(\frac{L}{2} - \frac{a}{4}\right) = \frac{P}{16L}(2L - a)^2$$

So for a 1.8m plank and typical skate board (a=65cm) carrying a 65kg person,

$$\hat{M}_C = \frac{65 \times 9.81}{16 \times 3}(3.6 - 0.65)^2 = 116\,\text{Nm}$$

If the plank is 13cm wide and 1.8cm thick, then the maximum bending stress is

$$\sigma = \frac{\hat{M}_C y}{I} = \frac{\hat{M}_C (h/2)}{(bh^3/12)} = \frac{6 \times 116}{0.13 \times 0.018^2} = 16.5\,\text{MPa}$$

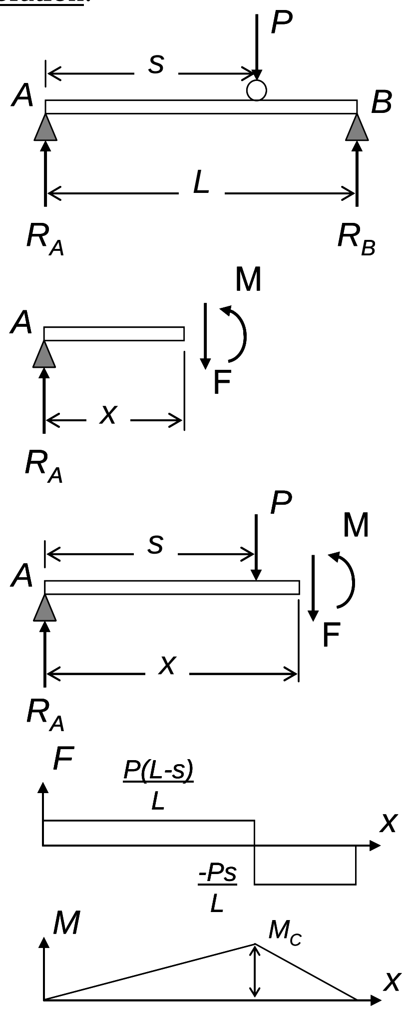
This compares to compressive ultimate strengths for common woods in the range 35 to 55 MPa parallel to the grain and 4 to 10MPa perpendicular to the grain.

Evaluate

<u>*Example 8.1*</u>

Ask students to repeat the analysis above but for unicyclist crossing the plank.

<u>Solution</u>:



Considering the complete beam

Resolve vertically: $R_A + R_B = P$

Moments about A: $Ps - R_B L = 0$

Thus: $R_A = \dfrac{P(L-s)}{L} \quad R_B = \dfrac{Ps}{L}$

Considering the cut section (0<x<s)

Resolve vertically: $F = R_A = \dfrac{P(L-s)}{L}$

Moments: $M = R_A x = \dfrac{P(L-s)x}{L}$

Considering the cut section (s<x<L)

Resolve vertically:

$$F = R_A - P = \frac{P(L-s)}{L} - P = -\frac{Ps}{L}$$

Moments:

$$M = R_A x - P(x-s) = \frac{Ps}{L}(L-x)$$

For maximum bending moment:

$$\frac{\partial M_C}{\partial s} = 0 \text{ and } M_C = R_A x = \frac{P(L-s)s}{L} = \frac{P}{L}\left(Ls - s^2\right)$$

So, $\dfrac{\partial M_C}{\partial s} = L - 2s = 0$ and $s = \dfrac{L}{2}$

Thus, $\hat{M}_C = \dfrac{P}{L}\left(\dfrac{L^2}{2} - \dfrac{L^2}{4}\right) = \dfrac{PL}{4} = \dfrac{65 \times 9.81 \times 1.8}{4} = 287\,\text{Nm}$

$$\sigma_{\max} = \frac{\hat{M}_C y}{I} = \frac{\hat{M}_C (h/2)}{(bh^3/12)} = \frac{6 \times 287}{0.13 \times 0.018^2} = 41\text{MPa}$$

Maximum stress of 41MPa induced when unicyclist at the middle. The position could have been deduced without analysis.

Example 8.2

Ask students to look for two other examples in their everyday life and explain how the above principles apply to each example.